King Fahd University of Petroleum & Minerals
Department of Information and Computer Science

Sample Solution

Question	1	2	3	4	5	6	7	8	9	10	Total
Max	20	7	7	10	7	7	7	7	14	14	100
Earned											

Question 1: [20 Points] [CLO 1, 2] The Foundations: Logic and Proofs
Indicate whether the given statement is true or false. In the answer column, write either **T** for "true" or **F** for "false". [-2 points for each incorrect answer].

Statement	Answer
1. Proofs are used to verify that computer programs produce the correct output for all possible input values, to show that algorithms always produce the correct result, to establish the security of a system, and to create artificial intelligence.	T
2. The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.	T
3. The basic building blocks of logic are propositions.	T
4. "q when p" and "p only if q" are equivalent ways to express $p \rightarrow q$.	T
5. A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.	T
6. Logic can be used to analyze and solve many familiar puzzles.	T
7. A compound proposition that is always false, no matter what the truth values of the propositional variables that occur in it, is called a contradiction.	T
8. The use of quantifiers enable us to reason with statements that assert that a certain property holds for all objects of a certain type and with statements that assert the existence of an object with a particular property.	T
9. Predicates are statements involving variables.	T
10. $\forall x \exists y P(x, y)$ is equivalent to $\exists x \forall y P(x, y)$.	F
11. Proofs in mathematics are arguments (valid and invalid) that establish the truth of mathematical statements.	F

Question 2: [7 Points] [CLO 1] Propositional Logic

Construct a truth table for the compound propositions: $\neg p \oplus \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \oplus \neg q$
F	F	T	T	F
F	T	T	F	T
T	F	F	T	T
T	T	F	F	F

Question 3: [7 Points] [CLO 1] Propositional Logic

State the contrapositive of the conditional statement: "If there is a quiz today, I will not attend tomorrow".

If I will attend tomorrow, there is not a quiz today.

Question 4: [10 Points] [CLO 1] Applications of Propositional Logic

Show solution steps to find whether these system specifications are consistent.

- Whenever the system software is being upgraded, users cannot access the file system.
- If users can access the file system, then they can save new files.
- If users cannot save new files, then the system software is not being upgraded.

Let:

u: "The software system is being upgraded."

a: "Users can access the file system."

s: "Users can save new files."

- Whenever the system software is being upgraded, users cannot access the file system.

  $u \to \neg a$

- "If users can access the file system, then they can save new files.

  $a \to s$

- If users cannot save new files, then the system software is not being upgraded.

  $\neg s \to \neg u$

One of these cases is enough to show that the system is consistent:

- **When *u* is FALSE, *a* is FALSE, and *s* is TRUE the system will be consistent.**
- **When *u* is FALSE, *a* is FALSE, and *s* is FALSE the system will be consistent.**
- **When *u* is FALSE, *a* is TRUE, and *s* is TRUE the system will be consistent.**
- **When *u* is TRUE, *a* is FALSE, and *s* is TRUE the system will be consistent.**

Question 5: [7 points] [CLO 1] Propositional Equivalences

Determine whether $(\neg r \wedge (s \rightarrow r)) \rightarrow \neg s$ is a tautology.

r	s	$\neg r$	$s \rightarrow r$	$\neg r \wedge (s \rightarrow r)$	$(\neg r \wedge (s \rightarrow r)) \rightarrow \neg s$
F	F	T	T	T	T
F	T	T	F	F	T
T	F	F	T	F	T
T	T	F	T	F	T

Therefore, it is a tautology.

Question 6: [7 points] [CLO 1] Predicates and Quantifies

Let $P(x)$ = "x is a baby" and $R(x)$ = "x is able to manage a crocodile"

Suppose that the domain consists of all people. Express the following statement using quantifiers; logical connectives; and $P(x)$ and/or $R(x)$.

Babies cannot manage crocodiles.

$\forall x(P(x) \rightarrow \neg R(x))$

Question 7: [7 points] [CLO 1] Predicates and Quantifiers

Rewrite the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$\neg \exists y(P(y) \wedge \forall x \neg Q(x, y))$

$\forall y \neg (P(y) \wedge \forall x \neg Q(x, y))$

$\forall y (\neg P(y) \vee \neg \forall x \neg Q(x, y))$

$\forall y (\neg P(y) \vee \exists x \neg (\neg Q(x, y)))$

$\forall y (\neg P(y) \vee \exists x Q(x, y))$

Question 8: [7 points] [CLO 1] Predicates and Quantifiers

Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, and 5. Express the following statement without using quantifiers, instead using only negations, disjunctions, and conjunctions.

$\neg \exists x P(x)$

$\neg (P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

$\neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4) \wedge \neg P(5)$

Question 9: [14 points] [CLO 2] Rules of Inference

Given $\neg p \land q$, $\neg r \rightarrow s$, $r \rightarrow p$, and $s \rightarrow u$, show that u.

[1] $\neg p \land q$	Premise
[2] $\neg r \rightarrow s$	Premise
[3] $r \rightarrow p$	Premise
[4] $s \rightarrow u$	Premise
[5] $\neg p \rightarrow \neg r$	contrapositive of [3]
[6] $\neg p$	Simplification of [1]
[7] $\neg r$	Modus ponens using [5] and [6]
[8] s	Modus ponens using [2] and [7]
[9] u	Modus ponens using [4] and [8]

Question 10: [14 points] [CLO 2] Introduction to Proofs

Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

We need to prove that:

- If n is odd if then $5n + 6$ is odd, and
- If $5n + 6$ is odd if then n is odd

Assume n is odd, so $n = 2k + 1$ for some integer k.

Then $5n+6 = 5(2k+1)+6 = 10k+5+6 = 10k+11 = 2(5k+5)+1$.

Thus, $5n+6$ is odd.

We now must prove the converse, "If $5n + 6$ is odd, then n is odd." For this, we will use proof by contrapositive. So the statement becomes "If n is not odd, then $5n + 6$ is not odd."

Assume that n is not odd, so $n = 2k$ for some integer k.

Then $5n + 6 = 5(2k) + 6 = 10k + 6 = 2(5k + 3)$.

Thus, $5n + 6$ is not odd.

Hence, we have proven that n is odd if and only if $5n + 6$ is odd.

RULES OF INFERENCE	
p ------ $\therefore\ p \vee q$	(Addition)
$p \wedge q$ ------ $\therefore\ p$	(Simplification)
p q ------ $\therefore\ p \wedge q$	(Conjunction)
p $p \rightarrow q$ -------- $\therefore\ q$	(Modus ponens)
$\neg q$ $p \rightarrow q$ -------- $\therefore\ \neg p$	(Modus tollens)
$p \rightarrow q$ $q \rightarrow r$ -------- $\therefore\ p \rightarrow r$	(Hypothetical syllogism)
$p \vee q$ $\neg p$ ------ $\therefore\ q$	(Disjunctive syllogism)
$p \vee q$ $\neg p \vee r$ ------ $\therefore\ q \vee r$	(Resolution)